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The determination of body shapes of minimum
drag using the Newton and the Busemann
pressure laws

According to the Newton and the Busemann formulae the pressure acting on the forward facing part of the body travelling in a gas may be explicitly expressed in terms of geometrical characteristics of the body surface. This enables to solve a variety of extremal problems for body shapes with minimum drag in steady motion.

In Ref.1 bodies of revolution of minimum drag have been found using the Newton formula with additional requirements that the body base diameter and the body length (or body surface area, or body volume) were given.

A number of papers (Ref.2-5) has been devoted to the more complicated extremal problem of finding airfoils and bodies of revolution of minimum drag using the Busemann formula.

The present report consists of two parts. The first part deals with statement and solution of extremal problem for airfoils and bodies of revolution with minimum drag using the Busemann pressure law. The second part is devoted to the determination of slender three-dimensional bodies with minimum drag using the Newton formula.

I. The determination of airfoils and bodies of
revolution with minimum drag using the Busemann formula.

This is the revised and improved text of previously published ^{papers} by the authors (Ref.2; Ref.4 - Chap.111, Sect.5, Ref.5).

2. The determination of three-dimensional
body shapes of minimum drag using the Newton
pressure law.

It is well known that at high speeds the pressure drag of bodies with unsymmetrical cross section may be lower than the drag of bodies of revolution having equal length and maximum

cross section. So in Ref.6 it has been shown that the drag of an

- 2 -

elliptical cone was less than the drag of an equivalent circular cone. This has been confirmed later by calculations made using the Newton pressure law and by experiments.

In particular the experiments carried out by authors with a set of elliptical cones in a wind tunnel at a Mach number 4 showed that the drag of an elliptical cone decreases with increasing eccentricity of its cross section. It is known also (see Ref.7) that at large speed the drag of a pyramidal body may be significantly less than the drag of an equivalent circular cone.

Below we discuss the determination of three-dimensional body shapes of minimum drag using the Newton pressure law

$$C_p = \kappa \cos^2(\bar{n}, \bar{U}) \quad (2.1)$$

Here $C_p = 2(p - p_\infty) / \rho_\infty U^2$ is the pressure coefficient, κ is a proportionality factor, \bar{n} - normal unit vector, \bar{U} - free stream velocity vector.

Let us assume the body surface in cylindrical coordinate system ρ, φ, z with z -axis in the flow direction ^(Fig. 2.1) to be given by the equation

$$\rho = z(\varphi) f(z) \quad (2.2)$$

If the length of the body is equal to unity and the function $f(z)$ is dimensionless one may put $f(1) = 1$.

In accordance with (2.1) and (2.2) the body drag coefficient (provided $f'(z) \geq 0$) may be found to be given by the following expression

$$C_D = \frac{\kappa}{S_1} \int_0^{2\pi} z^4(\varphi) \int_0^1 \frac{f(z) f'^3(z) dz}{1 + \frac{z'^2}{z^2} + z^2 f'^2(z)} d\varphi \quad (2.3)$$

Further we restrict the consideration to slender bodies only. Then in Eq.(2.3) the term $z^2 f'^2(z)$ may be dropped out and hence this equation can be simplified to the form

$$C_D = \frac{\kappa}{S_1} \int_0^1 f(z) f'^3(z) dz \int_0^{2\pi} \frac{z^4 d\varphi}{1 + \frac{z'^2}{z^2}} \quad (2.4)$$

This expression for the drag enables one to split the problem of finding the three-dimensional body having minimum drag into separate problems of determining the optimal shapes of its meridional contour and of its cross-sectional contour.

The optimum meridional contour for the Newton pressure law is well known and in this case $\phi \sim z^{3/4}$. In order to determine the cross-sectional contour one has to solve the following variational problem. Among the curves $z = z(\varphi)$ with a finite number of first derivative discontinuity ^{ies} points it is necessary to find the curve which corresponds to the lowest value of the functional

$$J = \int_0^{2\pi} \frac{z^4(\varphi) d\varphi}{1 + \frac{z'^2}{z^2}}$$

As additional requirements we prescribe the values of the maximum cross section area

$$S_1 = \frac{1}{2} \int_0^{2\pi} z^2(\varphi) d\varphi$$

and of its characteristic length z_0 .

The extremals of this problem must satisfy the Euler equation for the function

$$\mathcal{F} = \frac{z^4}{1 + \frac{z'^2}{z^2}} + \lambda^* z^2$$

Furthermore the Legendre condition $\mathcal{F}_{z''} \geq 0$ must be fulfilled along the extremal and at the points of discontinuity of the slope the Weierstrass-Erdmann conditions must be satisfied

$$(\mathcal{F} - z' \mathcal{F}_{z'})_{\varphi=0} = (\mathcal{F} - z' \mathcal{F}_{z'})_{\varphi=2\pi}, \quad (\mathcal{F}_{z'})_{\varphi=0} = (\mathcal{F}_{z'})_{\varphi=2\pi} \quad (2.5)$$

As the function \mathcal{F} does not contain the independent variable explicitly the corresponding Euler equation must be integrable by quadrature. By simple calculations we find the integral of this equation in the following form.

$$z_1^4 \left(1 + 3 \frac{z_1'^2}{z_1^2}\right) = (\pm 1 - \lambda z_1^2) \left(1 + \frac{z_1'^2}{z_1^2}\right)^2 \quad (2.6)$$

- 4 -

Here $\lambda = \lambda^* \sqrt{|C|}$; $\lambda_1 = \frac{\lambda}{\sqrt{|C|}}$ C is the integration constant. The equation (2.6) splits into two nonreducible equations of fourth order and in general it contains eight families of integral curves. Omitting the details of the qualitative analysis of this equation it must be noted that any extremal curve with recited features cannot belong only to one family of integral curves. Furthermore among eight families mentioned above four families cannot be extremals.

Let us transform Eq (2.6) to parametric form by taking as the parameter the quantity t given by $t = \frac{\lambda_1^2}{\lambda_1^2}$ (the index 1 will be next omitted). Then the remaining four families can be represented by following expressions

$$\lambda^2 = \frac{(-1)^i 2}{\lambda (1 + \sqrt{1 + (-1)^i \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}})} \quad d\varphi = \pm \frac{1}{2} t \frac{d\lambda^2}{\lambda^2} \quad i=0,1 \quad (2.7)$$

Hence it follows immediately that if $i=0$ then $\lambda > 0$ and if $i=1$ then $\lambda < 0$. Due to Legendre's condition the parameter t must vary between 0 and $\sqrt{3}$.

Integrating the Eqs. (2.7) with $\lambda > 0$ the solution in parametric form is found to be

$$\lambda^2 = \frac{2}{\lambda (1 + \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}})} \quad \varphi = \frac{2}{\lambda^2} \int_0^t \frac{(3-t^2)t^2 dt}{(1 + \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}}) \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}} (1+t^2)^3} \quad (2.8)$$

If $\lambda < 0$, then correspondingly

$$\lambda^2 = \frac{2}{\lambda (1 + \sqrt{1 - \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}})} \quad \varphi = \frac{2}{\lambda^2} \int_0^t \frac{(3-t^2)t^2 dt}{(1 + \sqrt{1 - \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}}) \sqrt{1 - \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}} (1+t^2)^3} \quad (2.9)$$

The expressions (2.8) and (2.9) determine only two families of integral curves. The remaining two families may be obtained by choosing the minus sign in the second of Eqs. (2.7).

The estimation of the angle φ shows that its maximal value is less than $\frac{\pi}{8}$. Hence the families of integral curves do not

contain any closed curves and all ^{integral} these curves lie within the ~~angle~~ angle determined by $0 \leq \varphi \leq \varphi_0 = \varphi(t_0)$

Now let us consider the third family of integral curves corresponding to $\lambda > 0$ and to the minus sign in the second of Eqs. (2.7). It can be easily seen that the variation of parameter t from t_0 to 0 gives the curves of the third family lying within the angle $\varphi_0 \leq \varphi \leq 2\varphi_0$.

Matching the curves of the first and of the third families gives the integral curves in the interval $0 \leq \varphi \leq 2\varphi_0$.

Actually the extension of the first family curve results in a curve symmetric to it with respect to the ray $\varphi = \varphi_0$. In order to extend the extremal curve for still larger angles we must again use the curve of the first family and repeat the above mentioned procedure. After a finite number of such steps we can obtain an extremal curve filling up an arbitrary large angle. If the angle 2π is multiple to φ_0 then the extremal curve will be closed. It is easy to prove that in junction points corresponding to $t = 0$ and $t = t_0$ the slope of the curves is discontinuous. Thus, in general the conditions (2.5) at those points will not be fulfilled. But the simple analysis shows that the first condition (2.5) is identically fulfilled at any junction point; the second condition is satisfied only at points where $t = 0$, that is at largest radius. If one choose the minimal radius as the characteristic length, then the variation of radius at $t = t_0$ would be equal zero and the second condition (2.5) is superfluous. Similarly, one can prove that the solution given by Eqs. (2.9) corresponds to the case when the maximal radius is chosen as characteristic length.

Now we shall deduce the relations determining the constants C, t_0 and λ . The requirement for the extremal to be closed

gives

$$\frac{2}{\lambda^2} \int_0^{t_0} \frac{(3-t^2)t^2 dt}{\left(1 + \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}}\right)^2 \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}} (1+t^2)^3} = \frac{\pi}{n} \quad (2.10)$$

where n is the number of double segments which the extremal curve consists of. The use of the isoperimetric condition and of the Eq.

(2.8) gives

$$\frac{2}{\lambda^2} \int_0^{t_0} \frac{(3-t^2)t^2 dt}{\left(1 + \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}}\right)^2 \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}} (1+t^2)^3} = \frac{S_1}{n\lambda^2} \frac{1}{\left(1 + \sqrt{1 + \frac{4t_0^2(3+t_0^2)}{\lambda^2(1+t_0^2)^2}}\right)} \quad (2.11)$$

Remembering that $\lambda_{min} = \lambda^0$ we find the third equation in the

form

$$2\sqrt{|C|} = \lambda^0 \lambda \left(1 + \sqrt{1 + \frac{4t_0^2(3+t_0^2)}{\lambda^2(1+t_0^2)^2}}\right) \quad (2.12)$$

The analysis of Eqs. (2.10) - (2.12) shows that they may be satisfied if n is sufficiently large. For fixed n the parameters λ and t_0 can be found from Eqs. (2.10) and (2.11) and the constant C - from Eq. (2.12). Hence we can find an infinite set of extremals each satisfying all requirements imposed by the statement of the variational problem. The shape of the cross-section is wholly determined by the parameters n and $\frac{S_1}{\lambda^0 \lambda^2}$, the value of λ^0 determines the size of the cross-section.

In accordance with Eqs. (2.4) the drag of optimal bodies is given by following expression

$$C_D = \frac{\kappa}{S_1} \int_0^1 \ell \ell'^3 dz. \quad \frac{4n\lambda^0 \lambda^4}{\lambda^2} \left(1 + \sqrt{1 + \frac{4t_0^2(3+t_0^2)}{\lambda^2(1+t_0^2)^2}}\right)^2 \int_0^{t_0} \frac{(3-t^2)t^4 dt}{\left(1 + \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}}\right)^3 \sqrt{1 + \frac{4t^2(3+t^2)}{\lambda^2(1+t^2)^2}} (1+t^2)^4} \quad (2.13)$$

The above expressions for the solution of the variational problem are rather complicated. Therefore let us consider the simplified expressions obtained in the limiting case $t_0 \ll 1$.

In this case Eqs. (2.8) - (2.13) may be transformed to following simplified expressions

$$\lambda^2 = \frac{2}{\lambda(1 + \sqrt{1 + z^2})} + O(t^4) \quad \varphi = \frac{\lambda}{4\sqrt{3}} \int_0^z \frac{z^2 dz}{(1 + \sqrt{1 + z^2}) \sqrt{1 + z^2}} + O(t^5) \quad z = \frac{\sqrt{2} t}{\lambda}$$

- 7 -

(2.14)

$$\frac{\lambda}{4\sqrt{3}} \int_0^{z_0} \frac{z^2 dz}{(1+\sqrt{1+z^2})\sqrt{1+z^2}} + O(t^5) = \frac{\pi}{n} \quad \frac{\lambda}{4\sqrt{3}} \int_0^{z_0} \frac{z^2 dz}{(1+\sqrt{1+z^2})^2 \sqrt{1+z^2}} + O(t^5) = \frac{S_1}{n z_0^2 (1+\sqrt{1+z_0^2})}$$

$$C_D = \frac{\kappa}{S_1} \int_0^1 \phi^3 dz \left[\frac{n z_0^4 \lambda^3}{6\sqrt{3}} (1+\sqrt{1+z_0^2})^2 \int_0^{z_0} \frac{z^4 dz}{(1+\sqrt{1+z^2})^3 \sqrt{1+z^2}} + O(t^7) \right]$$

All the integrals in Eqs. (2.14) can be expressed as simple functions.

The shape of the cross-section resulting from these equations is given by

$$z^2 = \frac{z_0^2 (1+\sqrt{1+z_0^2})}{1+\sqrt{1+z^2}} \quad \varphi = \frac{\lambda}{4\sqrt{3}} [z - \ln(z + \sqrt{1+z^2})] \quad 0 \leq z \leq z_0 \quad (2.15)$$

The values of the parametres are determined by following equations

$$\frac{\lambda}{4\sqrt{3}} [z_0 - \ln(z_0 + \sqrt{1+z_0^2})] = \frac{\pi}{n}$$

$$\frac{\lambda}{4\sqrt{3}} \left[\ln \left| \frac{1+z_0 + \sqrt{1+z_0^2}}{1-z_0 + \sqrt{1+z_0^2}} \right| - \frac{2z_0}{1+\sqrt{1+z_0^2}} \right] = \frac{S_1}{n z_0^2 (1+\sqrt{1+z_0^2})} \quad (2.16)$$

Note that, Eqs. (2.15) is written in terms of the original ^{first of} dimensional variables.

The drag coefficient in this limiting case can be expressed in the form

$$C_D = \frac{\kappa}{S_1} \int_0^1 \phi^3 dz = \frac{n z_0^4 \lambda^3}{6\sqrt{3}} (1+\sqrt{1+z_0^2})^2 \left[\frac{4z_0}{1+\sqrt{1+z_0^2}} - 3 \ln \left| \frac{1+z_0 + \sqrt{1+z_0^2}}{1-z_0 + \sqrt{1+z_0^2}} \right| + 2 \frac{z_0 (1+\sqrt{1+z_0^2})}{(1+\sqrt{1+z_0^2})^2 - z_0^2} \right] \quad (2.17)$$

The contours of the cross-section found using the above equations for $n = 10$ and $n = 15$ and for several values of $\frac{S_1}{z_0^2}$ are shown in Figs. 2.2 and 2.3. It is seen from these figures that the peaks forming the body surface elongate with increasing value of $\frac{S_1}{z_0^2}$. For the cases considered the value $\frac{z_{max}}{z_{min}}$ varies from about 2 to about 5. Thus the cross-section of the body likes a star.

- 8 -

The cross sections for $n = 10$ and $\frac{S_1}{\gamma^2} = 5,94; 9,34$ are shown in Fig. 2.4 and for $n = 12$ and $\frac{S_1}{\gamma^2} = 5,94; 7,72$ in Fig 2.5 .

The drag of these optimal three-dimensional bodies is found to be many times lower than the drag of the equivalent optimal bodies of revolution (having the same length and the same maximal cross section). It is easy to show using Eqs. (2.13) that the increasing number n of peaks reduces the drag and in the limit $n \rightarrow \infty$ the drag falls to zero. In the Fig. 2.6 the dependence of $\frac{C_D}{C_{Drev}}$ on $\frac{S_1}{\gamma^2}$ and on the number n is illustrated.

The statement that the drag of presented examples of optimal three-dimensional bodies is twenty or more times less than the drag of equivalent optimal bodies of revolution is of course too optimistic. The Newtonian flow near the concave part of the surface is physically not justified, so the more accurate approach to the problem taking in account the finite thickness of the gas layer between the body surface and the shock wave would lead to a not so strong decreasing of the drag. It is obvious from physical reasons that with n very large the actual drag of the body would be higher than the drag of the equivalent body of revolution.

The overall drag of bodies with great number of peaks and with large value of ^{the} parameter $\frac{S_1}{\gamma^2}$ will increase, also due to increasing skin-friction drag .

Hence the absolute minimum will apparently correspond to a not to great number of the peaks.

It should be noted that in accordance with the Newtonian concept the drag of the body remains unaltered if the sectors which the body consists of would be rearranged in other order. Hence we may construct a great number of various bodies of minimum drag from the one original body. From mathematical point of view these bodies will have contours with discontinuous dependence of radius on angle, from physical point of view they will have additional surface

- 9 -

of friction .

The above results permit easy to solve the variational problem of determining the minimum drag body with a plane side placed at zero angle to the stream direction. The solution follows immediately from symmetry principle and for $n=10$ the corresponding cross-section shape is shown in Fig. 2.7.

-10-

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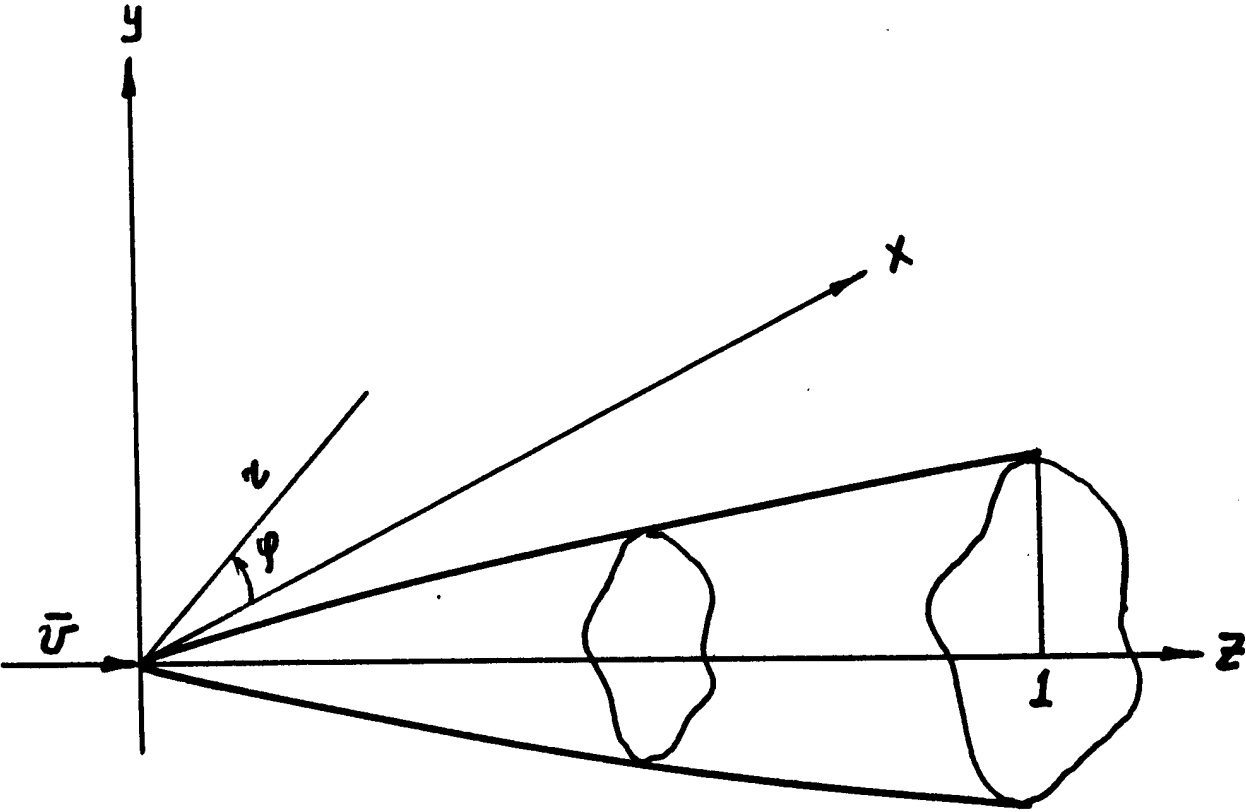


Fig. 2.1

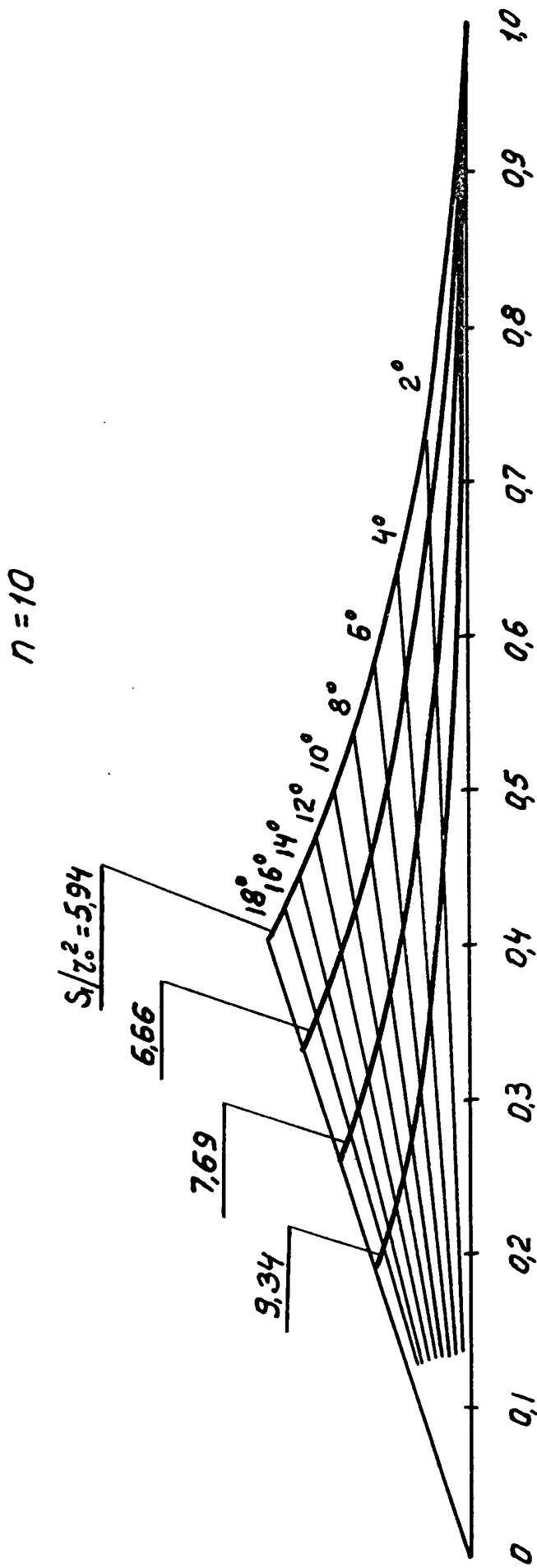


Fig. 2.2

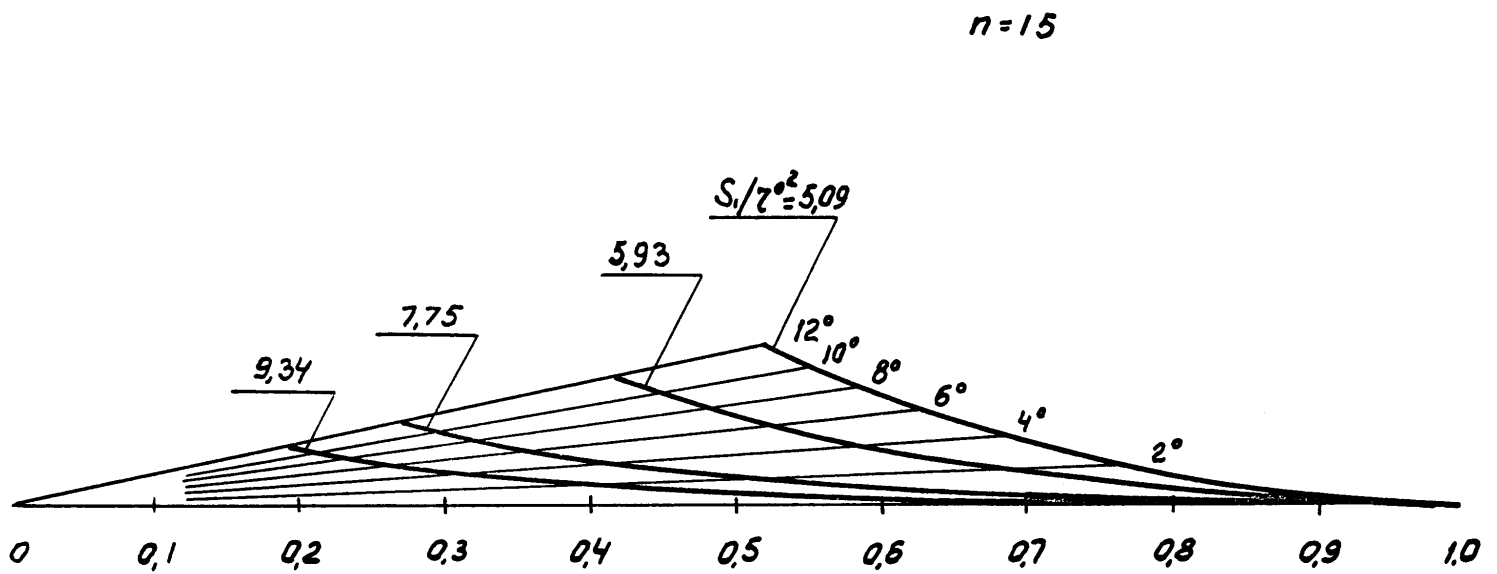


Fig. 2.3

$n=10$

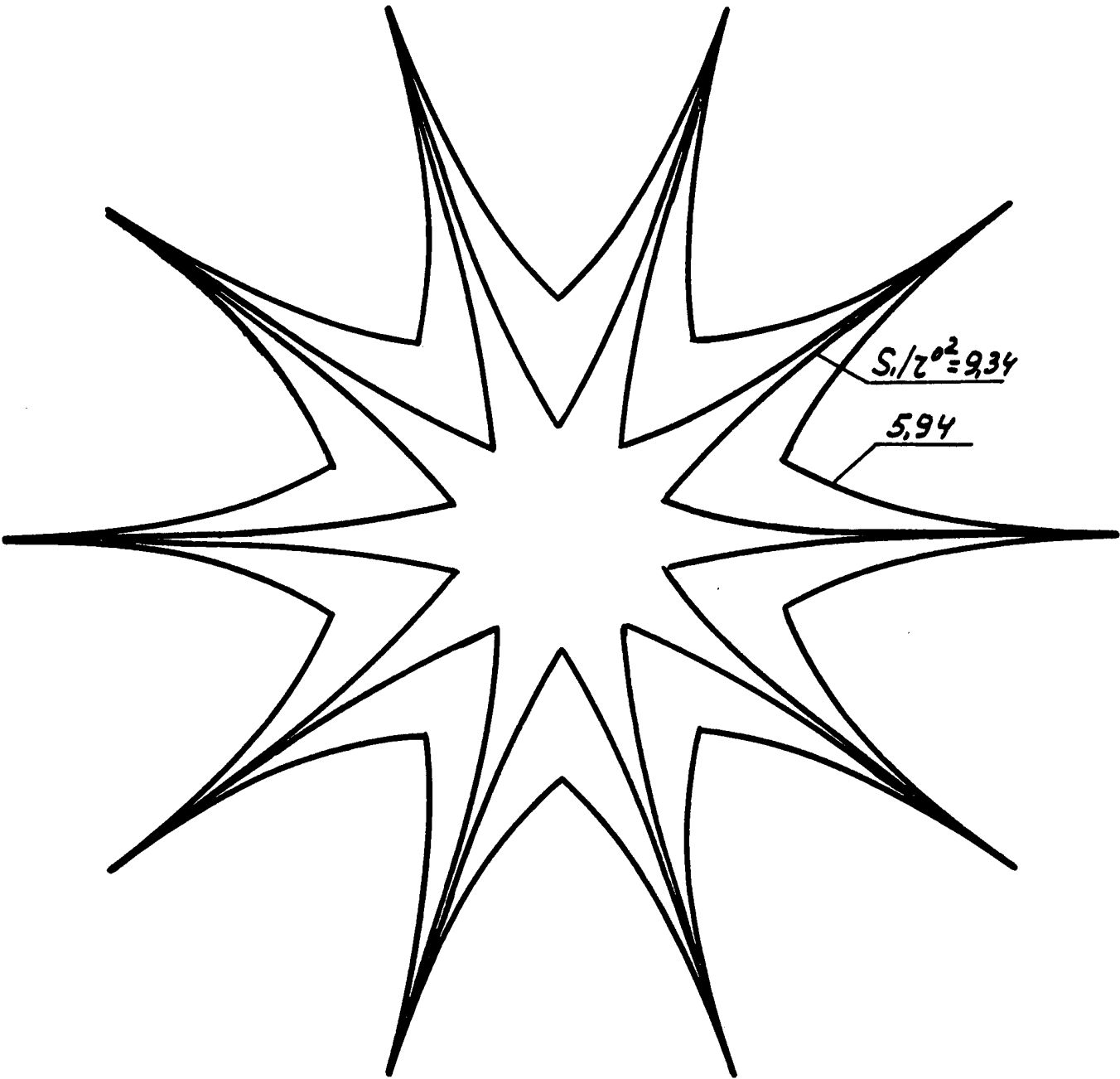


Fig. 2.4

$n=12$

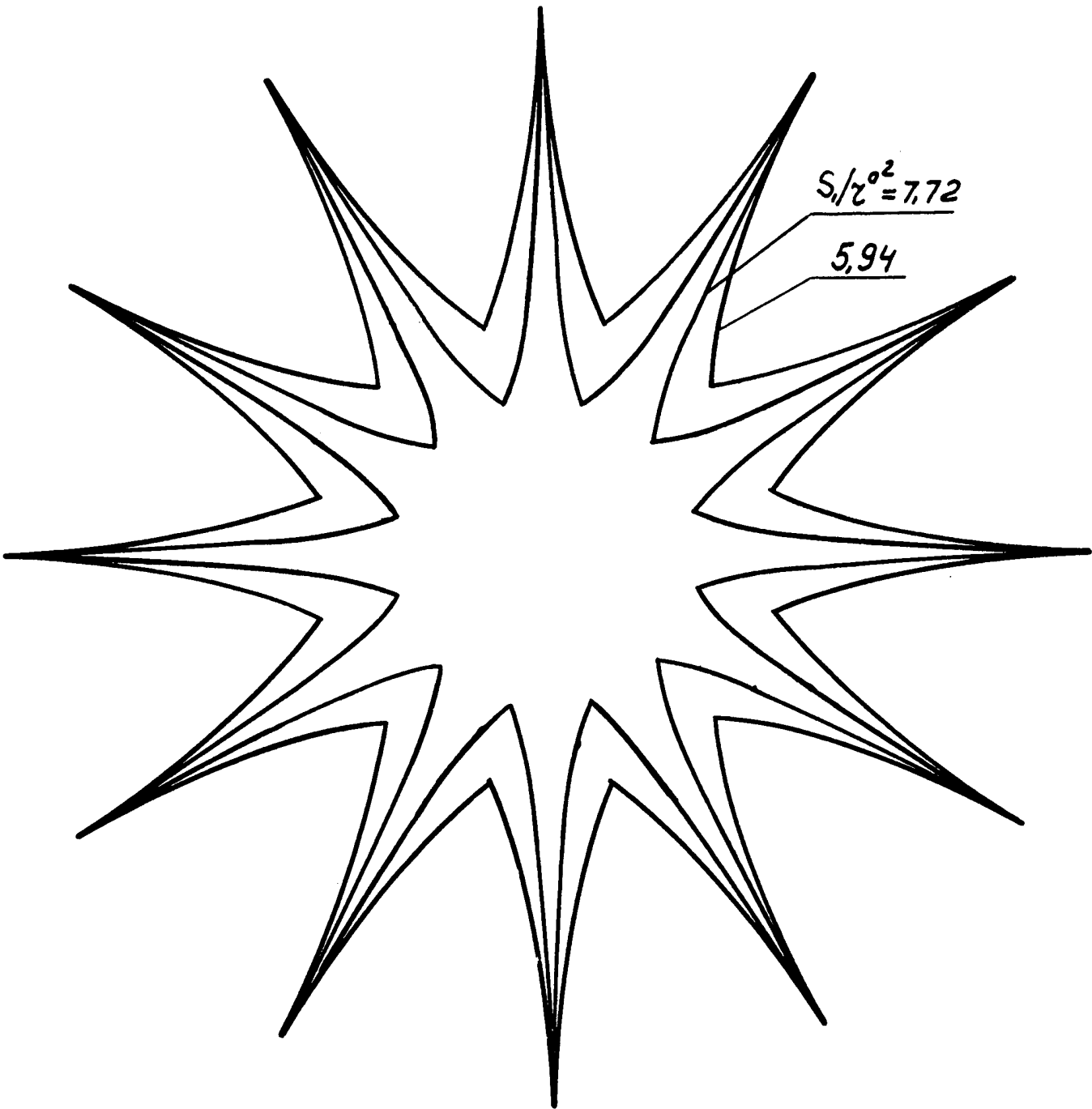
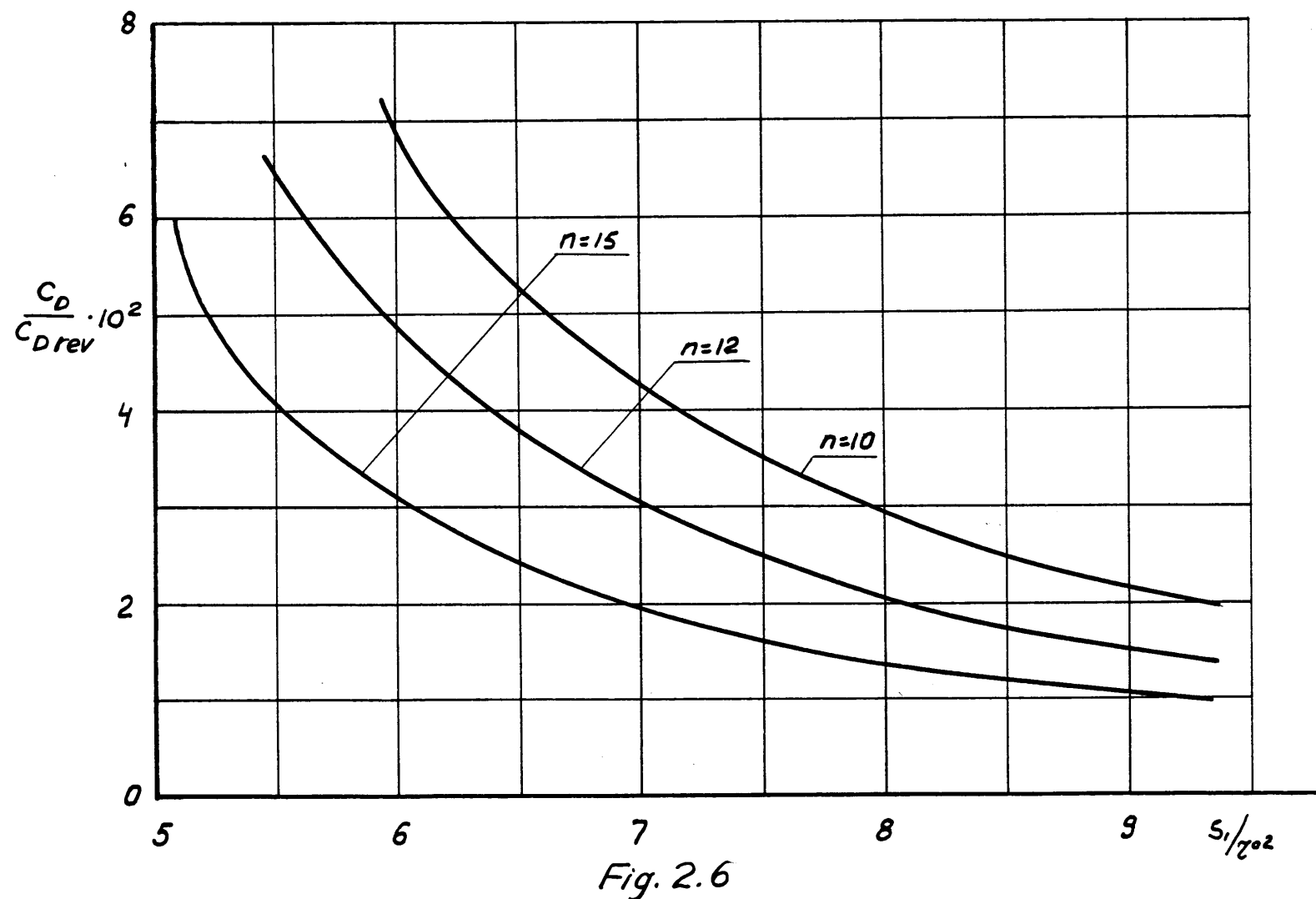


Fig. 2.5



$$n=10$$
$$S_1/z^{0.2} = 5.94$$

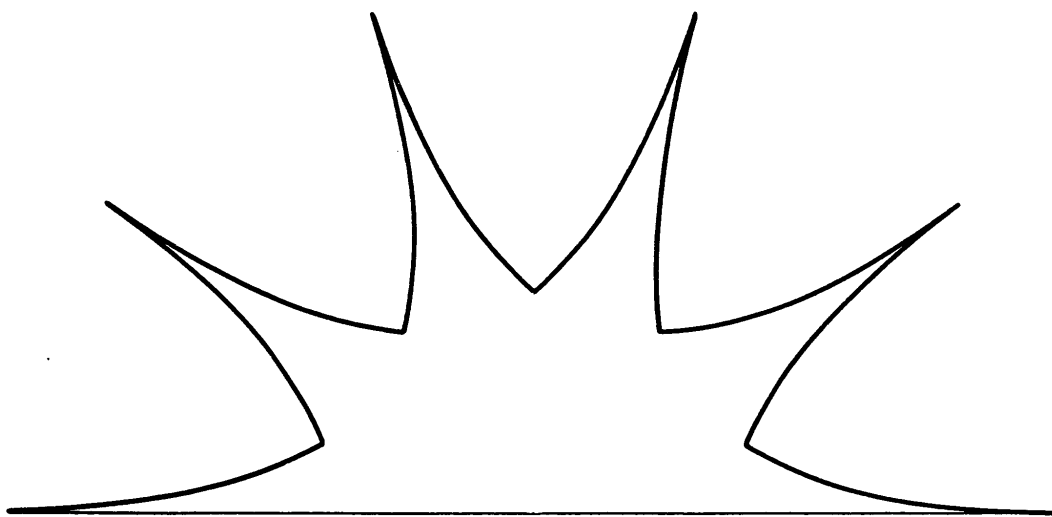


Fig. 2.7

- 1 -

COMMENTS ON THE PAPER entitled "The Determination of Body Shapes of Minimum Drag Using the Newton and the Busemann Pressure Laws" by G. G. Chernyi and A. L. Gonor, Moscow State University.

1. Professor Chernyi and Dr. Gonor have undertaken a difficult task, that is, to extend the existing knowledge on the theory of optimum bodies from the case where the independent variable is one to the case in which the independent variables are two. It is obvious that, if no simplifying assumptions are introduced, the Euler equations of this problem would be partial differential equations. However, by using the Newtonian pressure law, the slender body hypothesis, and by restricting the analysis to the class of homothetic bodies, the authors are able to reduce a problem in which the independent variables are two to two separate problems--each having a single independent variable. In other words, they are able to split the problem of the optimum longitudinal contour from that of the optimum transversal contour. This is a clever idea and the key point of the paper. However, such an idea involves a heavy price in terms of the

-2-

physical hypotheses and, hence, the immediate engineering usefulness of the paper.

2. The result that the absolute optimum body is one of zero pressure drag is certainly startling. However, its formal derivation does not even require the use of the formal methods of the Calculus of Variations and can be obtained as follows. Consider a circle of radius $r = \sqrt{S/\pi}$ and calculate the drag integral for such a circle. Next, consider a system of variations δr which are periodic with respect to φ and such that $\delta r/r \ll 1$ (Fig. 1). Assume that the period is $\Delta\varphi = 2\pi/n$ where n is the number of waves. Since the average value of r is unchanged, the isoperimetric condition is still satisfied. On the other hand, since $\dot{r} \neq 0$, the drag integral is smaller than the drag integral pertaining to the circle. As n increases, the drag integral decreases and, for $n \rightarrow \infty$, tends to zero which is the result obtained by Chernyi and Gonor.

3. In their paper, the authors state that the variational problem admits an infinite number of extremal arcs each of which is associated with a different number of segments of the starlike configuration and each of which satisfies the Euler equations, the corner conditions, and the Legendre

condition. Since the absolute minimum for the drag is achieved when

$n = \infty$, the following question arises: what is the meaning of the extremals associated with a finite number of n ? In all probability, ^{the} $n = \text{finite}$ ^{curves} are extremals only with respect to a system of weak variations (Legendre conditions) but not with respect to a system of strong variations (Weierstrass condition). However, it is probable that the curves $n \leq \text{finite}$ are extremal with respect to strong variations if an additional inequality constraint is imposed on the problem, that is, the constraint $n \leq \alpha$, where α is the upper bound to the number of segments composing the solution.

4. If the first integral of the problem of the transversal contour is written in the form

$$r^6 \frac{r^2 + 3\dot{r}^2}{(r^2 + \dot{r}^2)^2} + \lambda r^2 = C \quad (1)$$

it becomes obvious why the optimum configuration is starlike. Since Eq. (1) contains even powers of r and \dot{r} , it is clear that, for every solution of the form $r(\varphi)$, there exists another solution of the form $r(-\varphi)$. Also, for every solution $r(\varphi)$, there exist several solutions of the form $r(\varphi + \text{Const})$.

-4-

These two properties express the symmetry and the multiplicity of the solution and immediately lead to the idea that the configuration is starlike for $n = \text{finite}$.

5. The first integral (1) can be rewritten in the form

$$Ar^4 + Br^2 + C = 0 \quad (2)$$

where A, B, C depend on r, λ^*, C . This equation admits the solution

$$\frac{dr}{d\varphi} = \pm \sqrt{\frac{B \pm \sqrt{B^2 - 4AC}}{2A}} \quad (3)$$

whose variables can be separated as follows

$$d\varphi = \pm \sqrt{\frac{2A}{B \pm \sqrt{B^2 - 4AC}}} dr \quad (4)$$

Hence, the integration of the equations governing the extremal arc can be performed without recourse to the system of parametric representation employed by the authors.

6. It is probable that only one of the two signs preceding $\sqrt{B^2 - 4AC}$ is admissible. It is also probable that the Lagrange multiplier λ^* as well as the constant C are simultaneously positive.

-5-

7. It is difficult to imagine that a body of finite volume has zero drag. Hence, one wonders whether the results of the paper by Chernyi and Gonor could not be improved by a re-examination of the basic hypotheses, in particular, that concerned with the neglect of friction drag and that concerned with the pressure coefficient law.

8. In order to investigate to what extent, if any, the friction drag influences the solution of the problem considered by Chernyi and Gonor, the following analysis, based on direct methods, has been carried out at the Boeing Scientific Research Laboratories. The writer has considered the class of bodies (a) which are conical in the longitudinal sense and (b) which in the transversal sense include a basic circle of radius r_0 superimposed to which are n symmetric arcs of a logarithmic spiral satisfying the equation (Fig.2),

$$\dot{r}/r = \pm K \text{ where } K \text{ is a constant.}$$

Assuming that the minimum radius r_0 and the base area S are given, the analysis shows that the pressure drag D_p decreases monotonically with the number of segments n tending to zero for $n \rightarrow \infty$. On the other hand, the friction drag D_f increases monotonically with the number of segments tending to ∞ for $n \rightarrow \infty$. Consequently, the total (Fig.3),

$$\text{drag } D = D_f + D_p \text{ has a minimum with respect to } n.$$

As an example, if one

-6-

assumes $r_0 = 0.01$, $S = 0.07$, and $C_f = 1.6 \times 10^{-3}$, the optimum number of segments is $n \approx 8$; the drag coefficient C_D is 62% of the drag coefficient of the corresponding body of revolution C_{DR} . As another example, if one assumes $r_0 = 0.01$, $S = 0.07$, and $C_f = 3.2 \times 10^{-3}$, the optimum number of segments is $n \approx 6$; the drag coefficient C_D is 82% of the drag coefficient of the corresponding body of revolution C_{DR} .

9. Finally, a strong word of caution should be given with regard to Newton's pressure coefficient law employed by Chernyi and Gonor. As far as hypersonic aerodynamics is concerned, Newton's law must be regarded as an empirical law primarily valid for convex bodies only. Since Chernyi's optimal solutions are convex in the longitudinal sense but concave in the transversal sense, it seems possible that the optimistic results of the authors concerning drag reduction are mainly due to having applied an empirical law beyond the boundaries for which this law is intended. Consequently, the results of their paper should be interpreted as qualitative results confirming a well-known trend: that is, that bodies with a non-circular cross-section may exhibit a smaller wave drag than bodies with a circular cross-section. Quantitatively speaking, the authors' results are not valid for

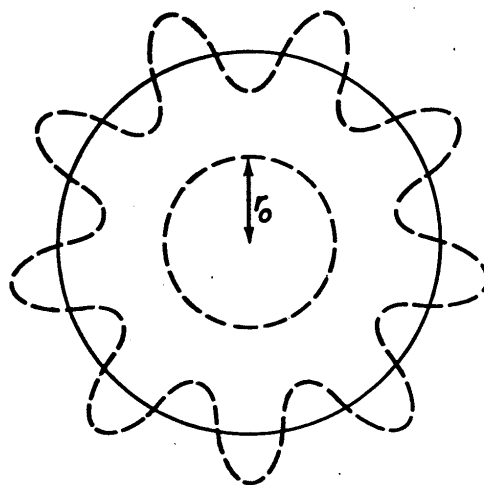
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bodies with a large number of segments n , even though they may be valid for

bodies with a small number of segments.

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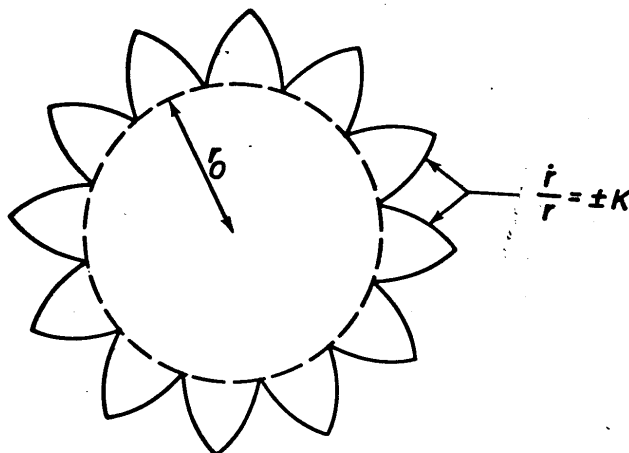
$$\int_0^{2\pi} \frac{r^6}{r^2 + i^2} d\phi \equiv \min, \quad \frac{1}{2} \int_0^{2\pi} r^2 d\phi \equiv \text{given}, \quad r > r_0$$



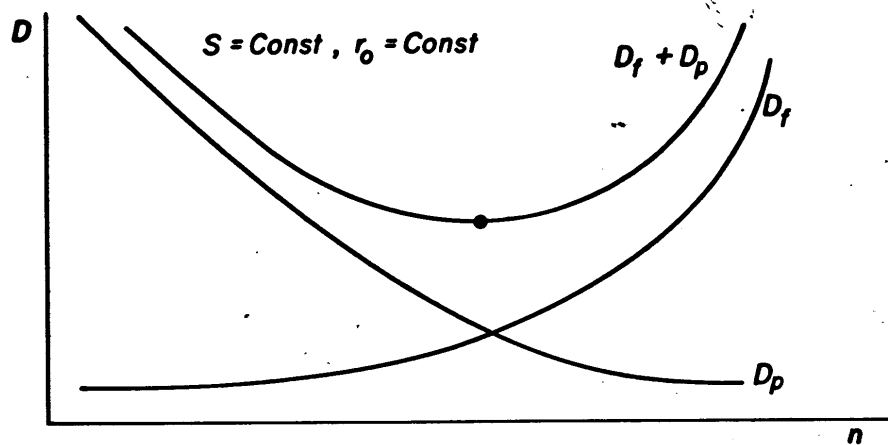
I

(a) Conical bodies

(b) Transversal contour: arcs of spiral superimposed on a circle



III



$r_0 = 0.10$
 $S = 0.07$

$C_f \times 10^3$	n	C_D / C_{DR}
1.6	8	0.62
3.2	6	0.82